

Scalar field description of decaying- Λ cosmologies

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The conditions under which cosmologies driven by time varying cosmological terms can be described by a scalar field coupled to a perfect fluid are discussed. An algorithm to reconstruct potentials dynamically and thermodynamically analogue to given phenomenological Λ models is presented. As a worked example, the deflationary cosmology, which evolves from a pure de Sitter vacuum state to a slightly modified FRW cosmology is considered. It is found that this is an example of nonsingular warm inflation with an asymptotic exponential potential. Differences with respect to other scalar field descriptions of decaying vacuum cosmologies are addressed and possible extensions are indicated..

I. INTRODUCTION

Phenomenological models with a time dependent cosmological “constant” $\Lambda(t)$ intend to explain how this term, or equivalently the vacuum energy density, reached its present value. Usually inspired by qualitative motivations, such proposals may indicate suggestive ways for solving the cosmological constant problem, as for instance, by describing the effective regimes that should ultimately be provided by fundamental physics (for reviews, see [1]). From a physical viewpoint, decaying vacuum models can also be attractive as a basis for a more realistic cosmology as suggested by the latest observations. These models are in line with recent measurements of luminosity distance based on SNe type Ia, which are consistently indicating the possible existence of an unknown form of energy with negative pressure, like an effective vacuum component, and presumably responsible for the present accelerating stage of the Universe [2].

Although incredibly small if compared to common microscopic scales, the cosmological Λ -term is expected to contribute dominantly to the total energy density of the universe. Moreover, since its present value, Λ_0 , may be a remnant of a primordial inflationary stage, it seems natural to study cosmological solutions including a decaying vacuum energy density which is high enough at very early times (to drive inflation), but sufficiently small at late times in order to be compatible with the present observations. A possible source for this effective cosmological term is provided by a scalar field (the so-called “quintessence”) which has received a great deal of attention lately [3], or still a true decaying vacuum energy density phenomenologically described by an equation of state $\rho_v(t) = -p_v(t)$.

In what follows, instead of a new decaying vacuum scenario, it is discussed how such phenomenological cosmologies can be interpreted in terms of a classical scalar field decaying into a perfect fluid. This

problem deserves particular attention because if a scalar field version of a $\Lambda(t)$ model can be implemented, its associated lagrangian can be used in other gravitation theories than general relativity. This is probably necessary if one wants to search for fundamental physics formulations of these models by considering effective high energy regimes in which general relativity is no longer valid (as in superstring cosmology, for example). Additionally, the methods employed here are specially addapted for a larger framework in which the dark energy is coupled to the dark matter, as suggested by Dalal *et al.* [4] (see also [5]). Such coupled scalar field models may avoid the cosmic coincidence problem, with the available data being used to fix the corresponding dynamics and, consequently, the scalar field potential responsible for the present accelerating phase of the universe. Another interesting feature of coupled dark energy models is that the temperature dependence on the redshift z of the relic radiation, $T(z)$, can be slightly different from the standard prediction deduced from the adiabatic FRW type expansion. Indirect measurements of $T(z)$ at high redshifts may become one of the most powerful cosmological tests because it may exclude the presence of a cosmological constant or even any kind of separately conserved quintessence [6].

This work is organized as follows. In Section II the dynamics and thermodynamics (along the lines of first order thermodynamics [7]) of time-varying Λ models are reviewed. A very simple procedure for describing such models in terms of scalar fields is proposed in section III. This algorithm, the most important result of this paper, incorporates thermodynamics into the pioneering treatment of Ellis and Madsen [8]. Applications of the procedure are discussed in section IV. Special attention is devoted to the deflationary $\Lambda(t)$ model suggested for the first time in Refs. [9], and for completeness, some of its variants recently presented in the literature have also been considered. Section V has a summary of the results and further applications of the methods used here are outlined.

II. TIME-VARYING COSMOLOGICAL TERM

Decaying vacuum cosmologies (see [1]) are described in terms of a two-fluid mixture: a decaying vacuum medium ($\rho_v = \Lambda(t)/8\pi G$, $p_v = -\rho_v$) plus a fluid component (the decaying vacuum products) described by their energy density ρ and pressure p . For the flat Friedmann-Robertson-Walker (FRW) line element ($c = 1$),

$$ds^2 = dt^2 - a^2(t) (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2), \quad (1)$$

where $a(t)$ is the scale factor. In such a background, the Einstein field equations (EFE) and the energy conservation law can be written as

$$8\pi G\rho + \Lambda = 3H^2, \quad (2)$$

$$8\pi Gp - \Lambda = -2\dot{H} - 3H^2, \quad (3)$$

$$\dot{\rho} + 3H(\rho + p) = -\frac{\dot{\Lambda}}{8\pi G} \equiv F, \quad (4)$$

where $H = \dot{a}/a$ is the Hubble parameter and F denotes a source term for the fluid with energy density ρ and pressure p . Notice that Bianchi identities (or Eq. (4)) impose that if the cosmological term is a time decreasing quantity, energy must be transferred from Λ to the perfect fluid. In order to keep the discussion as general as possible, it is usual to assume that the non-vacuum component obeys the γ -law equation of state

$$p = (\gamma - 1)\rho, \quad \gamma \in [1, 2]. \quad (5)$$

One may then use (2) and (3) to find the useful formula

$$\frac{2\dot{H}}{3H^2} = -\gamma_{\text{eff}}, \quad (6)$$

where

$$\gamma_{\text{eff}} = \gamma \left(1 - \frac{\Lambda}{3H^2} \right). \quad (7)$$

In terms of γ_{eff} , the matter-radiation energy density is given by

$$\rho = \frac{3H^2}{8\pi G} \frac{\gamma_{\text{eff}}}{\gamma}, \quad (8)$$

while the source term F appearing in (4) can be written as

$$F = 3H\gamma\rho \left(1 - \frac{\gamma_{\text{eff}}}{\gamma} + \frac{1}{3H\gamma} \frac{\dot{\gamma}_{\text{eff}}}{\gamma_{\text{eff}}} \right). \quad (9)$$

The above formulae identify the specific dynamics given for each different phenomenological $\Lambda(t)$ through the function $\gamma_{\text{eff}}(t)$. As will be seen later, for each phenomenological expression of $\Lambda(t)$ (see Table 1 of Ref. [1] for an extensive list of such models and respective references) the related γ_{eff} obtained from (7) may be transplanted to the analogue equations in the scalar field version. If one considers that there is no energy exchange between the ϕ field and the matter or radiation component, the associated potential can be found (at worst numerically) [8]. However, this case is not thermodynamically analogue to the pure $\Lambda(t)$ picture, as it presents particle production. In order to find the true equivalence, one has to fix the thermodynamics of the matter/energy creation process and impose the same thermodynamic conditions to both cases. In order to choose such conditions, it is worth to review the basics of the first order thermodynamics applied to vacuum decay.

The approach used here follows the lines set in Refs. [7,10]. First of all, it is assumed a continuous transfer of energy from the decaying vacuum to the γ -fluid, as given by Eq. (4). A more complete fluid description requires the definition of the particle current N^α and the entropy current S^α in terms of the fluid variables. If n denotes the number density of the created particles, the particle current is $N^\alpha = nu^\alpha$, and its balance equation can be written as

$$\frac{\dot{n}}{n} + 3H = \frac{\dot{N}}{N} \equiv \Gamma, \quad (10)$$

where Γ is the particle creation rate within a comoving volume. As a consequence of the vacuum “equation of state”, the entropy of the mixture depends exclusively on the matter component. It thus follows that the entropy current assumes the form

$$S^\alpha = n\sigma u^\alpha, \quad (11)$$

where σ is the specific entropy (per particle), and the mere existence of a nonequilibrium decay process means that $S^\alpha_{;\alpha} \geq 0$. Assuming local equilibrium, the thermodynamic variables are related by Gibbs' law:

$$nTd\sigma = d\rho - \frac{\rho + p}{n}dn, \quad (12)$$

where T is the temperature of the γ -fluid. From these relations the temperature evolution law for the fluid component is given as

$$\frac{\dot{T}}{T} = \left(\frac{\partial p}{\partial \rho}\right)_n \frac{\dot{n}}{n} + \frac{n\dot{\sigma}}{\left(\frac{\partial \rho}{\partial T}\right)_n}, \quad (13)$$

where

$$\dot{\sigma} = \frac{1}{nT}[F - (\rho + p)\Gamma]. \quad (14)$$

Note that if the usual equilibrium relations $n \propto T^{\frac{1}{\gamma-1}}$, and $\rho \propto T^{\frac{\gamma}{\gamma-1}}$ are valid, the specific entropy is constant ($\dot{\sigma} = 0$) and

$$\gamma\rho\Gamma = F, \quad (15)$$

as previously obtained for the “adiabatic” matter creation process [7,10]. This condition is widely used, specially for the radiation dominated phase, with $\rho \propto T^4$ and $n \propto T^3$. This is the thermodynamic constraint that will be fixed for the decaying Λ case and will also be used in its scalar field version [11]. In particular, this means that the entropy variation rate of the fluid (in a comoving volume) is

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N} = \frac{F}{\gamma\rho}. \quad (16)$$

It is worth to notice that the relation (15) between the source of energy and the source of particles is independent of $\Lambda(t)$, of the Einstein equations themselves, and it is also valid for the matter dominated phase. Using (9), (10) and (13) one can find the temperature law for the “adiabatic” process:

$$\frac{\dot{T}}{T} = -3H\frac{\gamma-1}{\gamma} \left(\gamma_{\text{eff}} - \frac{1}{3H} \frac{\dot{\gamma}_{\text{eff}}}{\gamma_{\text{eff}}} \right). \quad (17)$$

A systematic procedure for obtaining scalar field cosmologies analogue to given $\Lambda(t)$ models is proposed in the next section.

III. SCALAR FIELD DESCRIPTION

Even when phenomenologically well motivated (as in, for example, [12]), it is most desirable to have a derivation of time-varying Λ models from fundamental physics. Indeed, there are a few examples of dynamical Λ obtained as a result of fundamental processes. For instance, in Ref. [13], the back reaction of density perturbations on the de Sitter spacetime induces a varying vacuum energy density. In [14],

a relaxing contribution to the cosmological term comes from string motivated D -particle recoil effects. Although promising, these models are not complete, and some other possibilities might be attempted. In the former example, only the first stages of a genuine decaying vacuum cosmology are described, and in the latter, not all string-theory contributions to the vacuum energy were taken into account.

One way to seek for physically motivated models is to try to represent them in a field theoretical language, the easiest way being through scalar fields. If one is able to find a scalar field counterpart for a particular $\Lambda(t)$ version, it is natural to extend the model to other spacetimes and other gravitational theories, like an effective high energy string cosmology, for example. Another advantage is the possibility of quantizing the scalar field, which can help to find a fundamental justification for the phenomenological model. The procedure presented here works for a real and minimally coupled scalar field and generic $\Lambda(t)$ models, but it can be generalized for other cosmologies.

Following standard lines, the vacuum energy density and pressure into Eqs. (2) and (3) are replaced by the corresponding scalar field expressions, that is, $\frac{\Lambda}{8\pi G} \rightarrow \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$ and $-\frac{\Lambda}{8\pi G} \rightarrow p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$. The resulting EFE equations are:

$$3H^2 = 8\pi G \left(\frac{\dot{\phi}^2}{2} + V(\phi) + \tilde{\rho} \right), \quad (18)$$

$$3H^2 + 2\dot{H} = -8\pi G \left(\frac{\dot{\phi}^2}{2} - V(\phi) + \tilde{p} \right), \quad (19)$$

$$\dot{\tilde{\rho}} + 3\gamma H \tilde{\rho} = -\dot{\phi}(\ddot{\phi} + 3H\dot{\phi} + V'(\phi)) \equiv \tilde{F}. \quad (20)$$

where a tilde is used in the fluid component quantities in order to distinguish their values from their $\Lambda(t)$ counterparts. As remarked before, this is necessary because although dynamically equivalent (that is, having the same γ_{eff}) the two versions are, in principle, thermodynamically different. It can be seen that the above equations imply that the dynamic parameter

$$\gamma_{\text{eff}} = \frac{\dot{\phi}^2 + \gamma \tilde{\rho}}{\tilde{\rho}_t}, \quad (21)$$

where $\tilde{\rho}_t = \frac{\dot{\phi}^2}{2} + V(\phi) + \tilde{\rho}$ is the total energy density.

Now, in order to separate the scalar field contributions, it is interesting to introduce a second dimensionless parameter [15]

$$x \equiv \frac{\dot{\phi}^2}{\dot{\phi}^2 + \gamma \tilde{\rho}}, \quad (22)$$

which may be understood as follows. In order to evaluate how the potential energy V is distributed, it is convenient to compare the kinetic term $\dot{\phi}^2 = \rho_\phi + p_\phi$ with a quantity involving the energy of the material component. For the mixture, one such a quantity is $\tilde{\rho}_t + \tilde{p}_t = \dot{\phi}^2 + \gamma \tilde{\rho}$, which involves the redshifting terms of the Friedmann equation. As a measure of the relative weights of the redshifting components, x indirectly quantifies the amount of energy that the potential $V(\phi)$ is delivering to each component along the universe evolution, and as such, x is a quantity dependent on the thermodynamic

conditions underlying the decaying process of the scalar field. Perhaps not less importantly, x simplifies considerably the subsequent equations, since each part appearing in $\tilde{\rho}_t$ can be rewritten in terms of x and γ_{eff} ,

$$\tilde{\rho} = \frac{3H^2}{8\pi G} \frac{\gamma_{\text{eff}}}{\gamma} (1-x) = \rho(1-x), \quad (23)$$

$$\dot{\phi}^2 = \frac{3H^2}{8\pi G} \gamma_{\text{eff}} x, \quad (24)$$

$$V(H(\phi)) = \frac{3H^2}{8\pi G} \left[1 - \gamma_{\text{eff}} \left(\frac{x}{2} + \frac{1-x}{\gamma} \right) \right], \quad (25)$$

thereby allowing a direct comparison with the related quantities of the dynamic $\Lambda(t)$ case.

It should be stressed that the fluid component generated by the decaying scalar field is different from the one created by the decaying cosmological term. Besides, the mathematical problem has been restated in such a manner that, given γ_{eff} and x , one has the evolution of the “three” effective components, namely, $\tilde{\rho}$, $\dot{\phi}^2$, and $V(\phi)$. Now, assuming that both parameters are functions of time, or equivalently, of the scale factor or of the Hubble parameter, it can be shown that ϕ is given by one of the following forms:

$$\phi - \phi_I = \pm \sqrt{\frac{3}{8\pi G}} \int_{t_I}^t \sqrt{\gamma_{\text{eff}} x} H dt, \quad (26)$$

$$= \pm \sqrt{\frac{3}{8\pi G}} \int_{a_I}^a \sqrt{\gamma_{\text{eff}} x} \frac{da}{a}, \quad (27)$$

$$= \pm \frac{1}{\sqrt{6\pi G}} \int_H^{H_I} \sqrt{\frac{x}{\gamma_{\text{eff}}}} \frac{dH}{H}, \quad (28)$$

where (6) was used in the last expression. In principle, once $\phi(t)$, $\phi(a)$ or $\phi(H)$ is found, the inversion of the resulting expression can be done to get an explicit form for $V(\phi)$. Of course, if the expression is not directly invertible, a parametric reconstruction of the potential can still be obtained. As might be expected, two extreme and somewhat trivial situations arise naturally: (i) For $x = 0$ it is seen from (22) that $\dot{\phi}^2 = 0$, thereby recovering exactly the original $\Lambda(t)$ scenario (ii) If $x = 1$ one obtains $\tilde{\rho} = 0$ which leads to the limit of a pure scalar field evolution, i.e., the universe evolves with no matter, which is the situation originally dealt by Ellis and Madsen [8]. Nontrivial and physically interesting scenarios request an intermediary value of the x parameter ($0 < x < 1$).

Generically, for a given $\Lambda(t)$, the γ_{eff} parameter is obtained by using (7) and H is found by solving (6), but to compute the remaining functions, x must be specified. This can be done by imposing to the scalar field picture the same thermodynamic conditions applied to the $\Lambda(t)$ model, including the “adiabatic” particle production (see, for instance, [11]). In this case, the scalar field thermodynamics with a dissipative term is the same used in the last section (assuming that a classical scalar field do not carry entropy), with two important exceptions: the source term in the energy conservation equation

$$\tilde{F} = 3H\gamma\tilde{\rho} \left[1 - \frac{\gamma_{\text{eff}}}{\gamma} + \frac{1}{3H\gamma} \left(\frac{\dot{\gamma}_{\text{eff}}}{\gamma_{\text{eff}}} - \frac{\dot{x}}{1-x} \right) \right], \quad (29)$$

and the temperature law

$$\frac{\dot{\tilde{T}}}{\tilde{T}} = -3H \frac{\gamma-1}{\gamma} \left[\gamma_{\text{eff}} - \frac{1}{3H} \left(\frac{\dot{\gamma}_{\text{eff}}}{\gamma_{\text{eff}}} - \frac{\dot{x}}{1-x} \right) \right]. \quad (30)$$

Note, however, that the functional relation between the particle number and energy source terms is exactly the same (see Eq. (15))

$$\tilde{\Gamma} = \frac{\tilde{F}}{\gamma \tilde{\rho}}. \quad (31)$$

As it appears, a scalar field cosmology can be considered as equivalent, or at least analogue to a dynamic Λ version if they have the same dynamics (parametrized by γ_{eff}) and the same temperature evolution law. By comparing Eqs. (17) and (30) one can see that this constraint is satisfied for $x = \text{const}$. This is quite convenient, since the integrals (26)-(28) depend only on γ_{eff} under such a condition. As one may check, a constant x also implies that

$$\frac{\dot{S}}{S} = \frac{\dot{\tilde{S}}}{\tilde{S}}, \quad \frac{\dot{\rho}}{\rho} = \frac{\dot{\tilde{\rho}}}{\tilde{\rho}}, \quad \Gamma = \tilde{\Gamma}, \quad (32)$$

for any dynamics, thereby reinforcing this interpretation of thermodynamic analogy. In this context, x is a new phenomenological parameter that must be limited or obtained from cosmological data. This is consistent with the fact that scalar field cosmologies have one extra degree of freedom since there are two functions (the potential and the kinetic term) to be determined, instead of one, $\Lambda(t)$, in the decaying vacuum picture. The usefulness of the procedure outlined above may be better evaluated by working on particular models.

In order to make the basic steps of the algorithm more explicit, it may be pedagogical to apply it in the simplest possible situation. The most straightforward model to be considered is the one of Freese *et al.* [16] (see also [21,22]), which is characterized by the function

$$\Lambda(H) = 3\beta H^2, \quad (33)$$

where the dimensionless β parameter is contained on the interval $[0, 1]$. Inserting this expression into (7), one gets $\gamma_{\text{eff}} = \gamma(1 - \beta)$. Integrating (28) and substituting the result in (25), one has

$$V(\phi) = V_I e^{-\lambda(\phi - \phi_I)}, \quad (34)$$

where it was assumed that $\phi > \phi_I$, $\lambda = \sqrt{3\pi G \gamma(1 - \beta)/2x}$, and V_I is the expression (25) with $H = H_I$. This potential was already considered in [15], and can be interpreted as a sort of “coupled quintessence” [5]. One might also notice that the exponential potential would be obtained even if $\beta = 0$ (no particle production, like in some quintessence models) and $x = 1$ (absence of particles, equivalent to power-law inflation, if $\gamma < 2/3$).

There are two comments that should be made about the above procedure. Occasionally, one may have to use the γ_{eff} obtained from (7) and solve (6) first, in order to get an explicit functional relation for H , and then proceed to the subsequent steps to find $V(\phi)$. Naturally, there will be cases in which the system of equations will not have exact solutions, so that numeric calculations should be needed. Another important point is that, although analogue, the scalar field picture presented here is not

an exact description of its Λ version. Despite having the same temporal rates and following the same dynamics, the thermodynamic quantities have their values parameterized by the constant x , if compared to their Λ picture counterparts. This result may have phenomenological consequences. For instance, the temperatures of these two pictures are related by

$$\tilde{T} = (1 - x)^{\frac{1}{4}} T, \quad (35)$$

so that physical processes like nucleosynthesis and matter-radiation decoupling happen at different times, and phenomenological bounds on decaying Λ cosmologies, like the nucleosynthesis bounds found in [17], should be reevaluated.

IV. A CASE STUDY: THE DEFLATIONARY UNIVERSE

In previous papers [9], it was proposed a phenomenological decaying- Λ law that yielded a nonsingular cosmological scenario of the deflationary type (as Barrow [18] termed the pioneering Murphy's model [19]). In this model, the cosmic history started from an instability of the de Sitter spacetime in the past infinity, and, subsequently, the universe evolved towards a slightly modified Friedmann-Robertson-Walker (MFRW) cosmology. The solution may be described analytically, and the transition from the de Sitter to the MFRW phase is continuous with the decaying vacuum generating all the matter-radiation and dark energy of the present day universe. Broadly speaking, it resembles the warm inflationary scenarios [20] since the particle production process occurs along the inflationary phase and a reheating phase is not necessary. In addition, the maximum allowed value for the vacuum energy density may be larger than its present value by about 120 orders of magnitude, as theoretically suggested.

The above discussion shows that the important question here is how the vacuum energy density decays in the course of the expansion. Note that for a two component fluid it is natural to introduce the parameter:

$$\beta \equiv \frac{\rho_v}{\rho + \rho_v} = \frac{\rho_v}{\rho_t}. \quad (36)$$

This means that $\rho_v = 3\beta H^2/8\pi G$ or equivalently $\Lambda(H) = 3\beta H^2$ as claimed by the authors of reference [21] using dimensional arguments (see also [16,22]). The first case analysed was β constant [16]. However, it has been shown that such a scenario does not started from a de Sitter universe [21] so that a deflationary scenario is absent. In this way, a description of an earlier inflationary period with no matter (de Sitter) characterized by a definite time scale H_I^{-1} is possible only if one consider a time-dependent β parameter, or equivalently, higher order terms of H in the expansion of $\Lambda(H)$. In order to show the plausibility of the expression proposed in [9], consider the latter approach through the expansion

$$\Lambda(H) = 3\beta H^2 + \delta H^3, \quad (37)$$

where δ is a dimensional parameter, $[\delta] = (\text{time})^{-1}$, which must be fixed by the time scale of deflation. As one may check, the de Sitter condition, $\max[\Lambda] = 3H_I^2$, implies that $\delta = 3(1 - \beta)H_I^{-1}$. Inserting this result into (37) the phenomenological law considered in Ref. [9] is obtained, namely:

$$\Lambda(H) = 3\beta H^2 + 3(1 - \beta) \frac{H^3}{H_I}, \quad (38)$$

or still,

$$\rho_v = \beta \rho_t \left(1 + \frac{(1 - \beta)}{\beta} \frac{H}{H_I} \right), \quad (39)$$

and comparing (39) with (36), it is seen that the fractional vacuum to total density ratio parameter is now a time-dependent quantity.

The arbitrary time scale H_I^{-1} characterizes the initial de Sitter phase, and, together with the β parameter, is presumably given by fundamental physics. At late times ($H \ll H_I$), the second term on the right hand side of (38) can be neglected. This means that the β coefficient measures the extent to what the model departs from the standard flat FRW cosmology at late stages. This model may also be viewed as an early phase of the decaying Λ scenario originally proposed in [16].

Now, inserting the expression of $\Lambda(H)$ into (7) the following expression for the γ_{eff} is obtained

$$\gamma_{\text{eff}} = \gamma(1 - \beta) \left(1 - \frac{H}{H_I} \right). \quad (40)$$

Note that for $H = H_I$ this equation describes the de Sitter space-time and gives the maximum value for the cosmological term, which corresponds to the value of Λ for the unstable de Sitter phase. For $H \ll H_I$ the model behaves like a MFRW model modified by the β parameter. Following [9], the transition from de Sitter to the MFRW phase is exactly described as

$$H = \frac{H_I}{1 + \frac{H_I}{H_0} \left(\frac{a}{a_0} \right)^{\frac{3\gamma(1-\beta)}{2}}}, \quad (41)$$

where the subscript 0 refers to present time quantities. Integrating the above equation, one gets

$$t = t_e + \frac{1}{H_I} \left[\ln \left(\frac{a}{a_e} \right) + \frac{2}{3\gamma(1 - \beta)} \frac{H_I}{H_0} \left(\frac{a}{a_0} \right)^{\frac{3\gamma(1-\beta)}{2}} \right], \quad (42)$$

where “e” denotes the end of inflation and it was assumed that $H_I \gg H_0$ and $a_0 \gg a_e$. Recently, the above solution has also been discussed in a similar context by Gunzig et al. [23] for the particular case with $\gamma = 4/3$ and $\beta = 0$.

As one may check, the matter-radiation energy density is given by

$$\rho = \frac{3(1 - \beta)}{8\pi G} H^2 \left[1 - \left(\frac{H}{H_I} \right) \right], \quad (43)$$

and it can be shown that its maximum value is

$$\rho_m = \frac{(1 - \beta)}{18\pi G} H_I^2. \quad (44)$$

Therefore, the fluid component starts with a zero value for $H = H_I$, grows until ρ_m (for $H_m = (2/3)H_I$) and decreases throughout a MFRW phase which dynamically resembles some recent dark energy models [3] with tracking solutions, but presenting particle production. The Hubble parameter at the end of inflation ($\ddot{a} = 0$) can be written as

$$H_e = H_I \left(1 - \frac{2}{3\gamma(1-\beta)} \right). \quad (45)$$

Now, since for the radiation dominated phase, $\rho \propto T^4$, and from (44), the maximum temperature reached by the gas is

$$T_m = \left(\frac{30\rho_m}{\pi g_*} \right)^{\frac{1}{4}}, \quad (46)$$

where g_* is the number of spin degrees of freedom of the fluid components. From (44) and (45)

$$H_e = \frac{3}{2}H_m \left(1 - \frac{2}{3\gamma(1-\beta)} \right), \quad (47)$$

so that $H_e < H_m$ for any values of γ and β except $\gamma = 2$ and $\beta = 0$. Generically, this means that the universe attains its maximum temperature before the end of inflation. In this connection, an interesting and careful thermodynamic analysis of deflationary models including other processes than vacuum decay can also be found in Refs. [24,25].

As argued in the last section, a scalar field equivalent to the model above outlined can be obtained by using a constant x and the deflationary expression for γ_{eff} given by Eq. (40). In such a case, the following analytical expression for the scalar field potential is found

$$V(\phi) = \frac{3H_I^2}{8\pi G} \frac{1 - \epsilon \tanh^2[\lambda(\phi - \phi_I)]}{\cosh^4[\lambda(\phi - \phi_I)]}. \quad (48)$$

for $\phi > \phi_I$, $\epsilon = (1-\beta)[1-x(1-\gamma/2)]$ and $\lambda = \sqrt{\frac{6\pi G\gamma(1-\beta)}{4x}}$. As one should expect, the above potential goes asymptotically to the exponential potential (34). Furthermore, it was obtained for a constant x , but if the required thermodynamic equivalence is relaxed, x can be a variable quantity, and different potentials can be found for the same deflationary behavior. Even the exponential potential would be obtained if, for instance, $x \propto \gamma_{\text{eff}}$. Of course, as can be seen from (29) or (30), in order to get a time-varying x one would have to provide appropriate thermodynamic conditions for the system.

Potentials with the functional form of (48) were also found by Maartens *et al.* [26] and by Zimdahl [24], but their scenarios have important differences with respect to the deflationary model studied here. In the first case, the authors considered $x = 1$ (no matter or radiation) and represented only the deflationary dynamics, with a transition from a de Sitter for a radiation-like dynamics driven solely by the interchange of energy between the ϕ potential and its kinetic term. Their potential is a particular case of the one presented here, with $\gamma = 4/3$, $x = 1$ and $\beta = 0$. Zimdahl obtains the same potential of Maartens *et al.* (and the same values for γ , β and x), but in a different thermodynamic context, so that it cannot be considered as a case of (48). Another important difference is that the particle creation rate $\tilde{\Gamma}$ in Zimdahl's work is not the same one of the deflationary $\Lambda(t)$ picture treated here, thereby violating one the equalities (32). Besides, the initial value for the fluid energy density in his picture is not zero, so that there is not a thermodynamic equivalence between the $\Lambda(t)$ and scalar field counterparts in the sense above discussed.

Another interesting feature of such models is that the scalar field and the fluid are thermally coupled during the inflationary era so that they can be regarded as instances of warm inflationary scenarios [20].

The connection between the two approaches is just the function \tilde{F} . In warm inflation, this thermal coupling is represented by a dissipative term $\Gamma_\phi \dot{\phi}^2$ in the energy conservation equation. As usually the adiabatic condition or, alternatively, the perfect fluid thermodynamic relation ($\tilde{\rho} \propto \tilde{T}^4$) is assumed, one may write Γ_ϕ in terms of the particle creation rate $\tilde{\Gamma}$:

$$\Gamma_\phi = \frac{1-x}{x} \tilde{\Gamma}. \quad (49)$$

For a constant x (that is, $\Gamma = \tilde{\Gamma}$), the above equation allows one to translate the results of a given dynamical $\Lambda(t)$ model to its warm inflationary counterpart. Furthermore, in such a case Γ_ϕ has the direct interpretation of a particle production rate (in a comoving volume). Since warm inflation does not have an uncontroversial derivation from first principles (see, for example [27]), the methods employed here might also be useful in the search for a more fundamental justification of warm scenarios.

V. CONCLUSION

A procedure to write scalar field versions of decaying vacuum cosmologies has been proposed. The switching between the two pictures depends basically on a pair of parameters (γ_{eff}, x) conveniently chosen. The first one, γ_{eff} , is responsible for the dynamic equivalence while the x parameter, measuring the fraction of energy carried by the redshifting components of the universe, can be related to the thermodynamic behavior. It has been shown that the two pictures can be made dynamically and thermodynamically equivalent in the sense that the cosmological quantities may have the same evolution laws. In a worked example (the deflationary cosmology), the time independent x parameter gave rise to an exponential potential for the scalar field, but other potentials can be found if x is allowed to vary in the course of the evolution.

It is worth mentioning that the set of equations presented in Section III is not necessarily limited to the search for scalar field counterparts of decaying vacuum scenarios. In principle, the procedure discussed here opens the possibility of scalar field versions involving other phenomenological descriptions, like bulk viscosity and matter creation cosmologies. In this connection, it is also interesting to investigate how the causal thermodynamic approach (or second order theories) [29] constrains the x parameter.

Finally, since a time-varying x can be used together with γ_{eff} as free parameters, this approach can be applied to solve “inverse problems” by using present and future observational data to reconstruct the potential $V(\phi)$ either by considering particle production [4] or universes containing a perfect fluid plus a decoupled scalar field [28]. Further investigations in this direction are in course.

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